

A PRACTICAL METHOD FOR THE CALCULATION OF FIRE RESISTANCE OF REINFORCED CONCRETE COLUMNS

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Abstract: In this paper a practical method for the determination of the fire resistance time of the uniaxially stressed reinforced concrete columns, which does not need the sophisticated computer aid, has been introduced.

Keywords: Fire resistance, Reinforced concrete column, Eurocode 2, Calculation Method

Betonarme Kolonların Yangın Dayanımının Saptanması için Pratik bir Hesap Yöntemi

Özet: Bu çalışmada, tek eksenli eğilmeye maruz betonarme kolonların yangın dayanımının hesabında, detaylı bilgisayar desteği gerektirmeyen bir yöntem tanıtılmıştır. Burada tanıtılacak olan yöntem ile yangın durumu için betonarme kolonların yangına dayanım süresi basit bir işlem ile belirlenebilmektedir. Hesap işlemleri virtüel iş yöntemini kullanmaktadır.

Anahtar Kelimeler: Yangın Dayanımı, Betonarme Kolon, Eurocode 2, Hesap Yöntemi

1. INTRODUCTION

Fire is one of the serious potential risks to most buildings and structures. The supporting columns to a building play a critical role in its stability under fire conditions. Progress in the field of theoretical prediction of fire resistance has been rapid in recent years. This was clearly demonstrated in a major research program at the special fire research department of the Technical University Braunschweig (SFB 148¹, Kordina et al., 1975). The research was mainly divided into two groups. The first group studied the changes in material properties of concrete at high temperatures (Schneider, U., 1972-1988, Haksever, 1986). The second group intensified on the structural performance of concrete elements under fire conditions (SFB 148 A, 1972-1988).

Reinforced concrete structural members (RFC) generally show good performance under fire situations. However, results from a number of studies (SFB 148, 1972-1988) have shown that (RFC) exhibit lower fire-resistance properties due to faster degradation of strength at elevated temperatures and fire-induced spalling as well. Thus, concern has developed regarding the determination of behavior of (RFC) experimentally in special designed furnaces. Such a method is time consuming and expensive. To overcome these disadvantages, considerable amount of research has been made to develop numerical methods. This procedure enables the behavior of a (RFC) to be predicted by much less expensive theoretical investigations (Upmeyer et al, 2008). Additionally design diagrams were developed which are similar to existing diagrams for ambient temperature conditions and can be applied to all typical types of cantilever columns (Hosser, D., Richter, E., 2009).

(RFC) are predominantly stressed by the compression forces. They are in buildings connected to other structural members, like beams, girders and slabs. If a building or part of it is exposed to a fire, the reinforced concrete columns experience in addition to direct thermal and mechanical stress attacks also second-order effects which determine finally the fire resistance of (RFC) (Klingsch et al, 1977). To build up a model to analyse the results of these effects, a much simplified method than the others (Quast et al, 2008) is developed and introduced in this paper.

2. FUNDAMENTALS OF THE CALCULATIONS

The fire resistance analysis of (RFC) constitutes an important part in their design. The numerical and reliable determination of the fire resistance time of reinforced concrete columns leads comparatively extensive computer calculations. The extensive calculations result in particularly from the time and temperature-dependent parameters, which cause locally different stress and deformation conditions in column behavior. The numerical analysis of a (RFC) in fire requires a sound theoretical model of the interaction between the fire and the structural member which is a very complex task. For this reason concern rose to develop a number of simplified models for the evaluation of the fire resistance of reinforced concrete columns e.g. in (Eurocode 2, EC2, 1997). It is however, still difficult to fulfill this task by a structural engineer in his/her practical, professional life even using these equations given in standards.

Thermal Analysis

The analysis for the fire behavior of (RFC) can be subdivided into two major sequences; a nonlinear temperature analysis and a successive structural

¹Sonderforschungsbereich 148, a special research activity for structural fire protection

analysis. Recent rapid progress in personal computer technology enables structural engineers to solve many complicated nonlinear problems by means of the different methods.

The temperature information of the (RFC) with time is essential for the successive solution of the structural analysis. In general, the heat flux from the fire to the structure at fire-side is governed by convection and radiation while the heat flow inside the element is determined by conduction. From the first law of thermodynamics and Fourier's law, the heat conduction within the material is represented as

$$\nabla^2 T(x, y, t) = \frac{1}{\alpha} \frac{\partial T(x, y, t)}{\partial t} \quad (2.1)$$

where k is the conductivity [W/m K], ρ the density of the material [Kg/m³], $\alpha = k / (\rho C_p)$ and C_p the specific heat of the material [J/kg K], α is the thermal diffusivity of the material [m²/s], T is the temperature distribution *within the structural member*. The internal heat generation has not been present in the investigations in Eq. 2.1 the standard fire exposure is described by an increasing temperature-time curve reads

$$\Delta T(t) = 20 + 345 \log_{10}(8t+1) \quad (2.2)$$

T represents the gas temperature *in the fire furnace*.

The transient thermal fields in the columns with rectangular cross-sections subjected to fire were calculated by the discrete element method in this paper (Kordina et al., 1975, Paulsson, 1983), which enables the effective prediction of thermal fields in concrete cross sections. The average steel bar temperature is approximately obtained by considering the column to consist entirely of concrete and the temperature at the centroid of each concrete element is selected as its representative temperature.

Structural analysis

Several methods are available for the determination of displacements, of which the most well-known is the method of virtual work. However, when the complete deformed shape of a structural member is required, the method of virtual work becomes rather laborious and is really unsuitable. In such a case, the other methods are more advantageous.

In (RFC) the axial forces are relatively high which causes a change in geometry and a change in stiffnesses along the column axis arising from the bending by the axial force. This effect means simply an increase in the amount of calculations making the use of computer necessary. However if the stiffness's are presented in tabular form for different load, cross-section and thermal combinations, calculations can be carried out then by a structural engineer with much less laborious procedures without any need to extensive computer aid (s. Table 1). The procedure is therefore to carry out the moment distribution with respect to the first-order theory, then, from the results to obtain the stiffness

distribution along the column axis and to determine the more accurate values of deflections with respect to the second-order theory until a stable equilibrium condition is attained (s. chapter 5).

Calculation of the warping of a concrete cross section

The nonlinear response of a cross-section "i" is dependent upon the acting internal forces and temperature distribution "T". It can be expressed by the function

$$\kappa_{p,T} = \kappa_{p,T}(\bar{p}, T)^{(i)} \quad [\bar{p}] = [\bar{M}, \bar{N}]^{(i)} \quad (2.3)$$

The curvature function $\kappa_{p,T}$ is generally nonlinear and can be determined iteratively by numerical procedures which provide sufficient accuracy. The $\kappa_{p,T}$ of the cross section can be written (s. Fig. 1) under acting loads and the fire action as

$$\kappa_{p,T} = \frac{\varepsilon_1 - \varepsilon_2}{d} \quad (2.4)$$

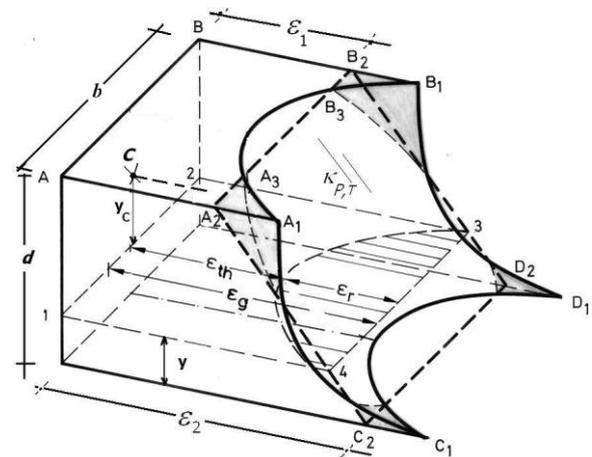


Fig. 1: Deformation and equilibrium condition of a cross section "i" under fire action.

In Fig. 1, the thermal expansion ε_{th} and the Bernoulli-Plane $\kappa_{p,T}$ are illustrated in the equilibrium condition for a cross section. $\varepsilon_r = \varepsilon_{th} - \varepsilon_g$ represents the cracked area in concrete while ε_g represents the total strain of the Bernoulli-Plane. Fig. 1 shows that the free thermal expansion is not uniform for the fire case related to the temperature distribution. Stress generating areas for compression are shaded on the figure. Stress calculations consider the nonlinear-nonreversible material response as the function of the acting temperature and stress distribution " $\sigma_t = \sigma(\varepsilon_r, T, \sigma_{t-1})$ ".

The strains $\varepsilon_1, \varepsilon_2$ at the upper and lower fibers of any cross section can be determined by the total differentials of the simultaneous functions in 2.5 and 2.6, (Haksever, 2012).

$$N = f_1(\varepsilon_1, \varepsilon_2) \quad (2.5)$$

$$M = f_1(\varepsilon_1, \varepsilon_2) \quad (2.6)$$

Definition of the effective flexural stiffness of a structural element

In order to use an appropriate simplification in the inelastic calculation, the flexural stiffness of the elements of a structural system is new defined and a *high-temperature effective flexural stiffness “(EI)_e”* of the structural members is adopted. The *(EI)_e* means that the nonlinear material behavior of reinforced concrete structural members (Haksever, 1982) as well as the acting internal forces are taken into account due to the high temperatures in the cross section.

The effective flexural stiffness “(EI)_e” can be determined for a cross section “i” as given in Eq. 2.7.

$$(EI)_e^{(i)} = \left[\frac{M}{\kappa_{P,T} - \kappa_T} \right]^{(i)} \tag{2.7}$$

In Eq. 2.7 “ κ_T ” shows the curvature solely

dependent of the temperature distribution by which the acting forces M_i and N_i are set equal to zero. In case of symmetry concerning the fire exposure and cross section κ_T is set also equal to zero.

3. STIFFNESS TABLES

One of the prepared stiffness tables of a concrete cross section is given below (Table 1). The reader can prepare these tables by himself easily.² Thus the axial force and the bending moment can be looked upon as parameters which affect the stiffness of the (RFC) section and once these have been determined, e.g. from Table 1, the methods of analysis of linear structures can be applied. For example, the reader can find from Table 1 for M=15 kNm, N=500 kN and $\Sigma A_s=1885 \text{ mm}^2$ ($6 \bar{\Phi} 20$) the stiffness, $EI=6.7890.10^{12} \text{ Nmm}^2$ for 30 min. standard fire duration. A linear interpolation is allowed between the M, N-combinations.

Table 1: Stiffness of a cross section 300/300 mm for different M, N combinations after an ISO830 fire exposure for 30 min. fire duration.

b/h = 300/300 mm, A _s = A _{s1} , f _{ck} =350, f _{yk} =4200 N/mm ²								
M[kNm] = 5.0 10.0 15.0 20.0 25.0 30.0								
N[kN], A _s [mm ²]		EI[Nmm ²] x 10 ¹²						t=30. Min.
300.	1206.	4.5551	4.6170	4.6247	4.6273	4.6298	4.6407	
	1527.	5.3019	5.3440	5.3529	5.3674	5.3781	5.3856	
	1885.	6.1571	6.1935	6.1811	6.2031	6.2177	6.2310	
400.	1206.	5.1544	5.0853	5.0264	4.9906	4.9372	4.9033	
	1527.	5.7559	5.7447	5.7099	5.6905	5.6577	5.6273	
	1885.	6.4638	6.4947	6.4969	6.4967	6.4661	6.4484	
500.	1206.	5.3869	5.3973	5.3238	5.2899	5.2333	5.1425	
	1527.	6.0887	6.0598	6.0108	5.9667	5.9337	5.9284	
	1885.	6.9147	6.8146	6.7890	6.7518	6.7207	6.6907	
600.	1206.	5.5075	5.5854	5.5531	5.4711	5.3613	5.2842	
	1527.	6.2410	6.1972	6.2115	6.2469	6.0834	5.9727	
	1885.	6.9816	6.9960	6.9851	6.9303	6.9173	6.8889	

4. CALCULATION OF THE FIRE RESISTANCE OF (RFC)

Deflections of (RFC)

Broadly speaking the failure of a (RFC) can occur under fire exposure by yielding of the material at a sufficient number of locations to form a mechanism or by steadily increasing deflections due to axial force without

the bearing capacity in column exceeding the load bearing capacity which leads to a stability failure.

The deflected shape for a prismatic (RFC) with pin-jointed supports is given in Fig. 2. The column in Fig. 2a can also be analyzed as the column with fixed-end support at the bottom and with joint translation at the top end (s. Fig. 2b).

²The computer program can be obtained from the author.

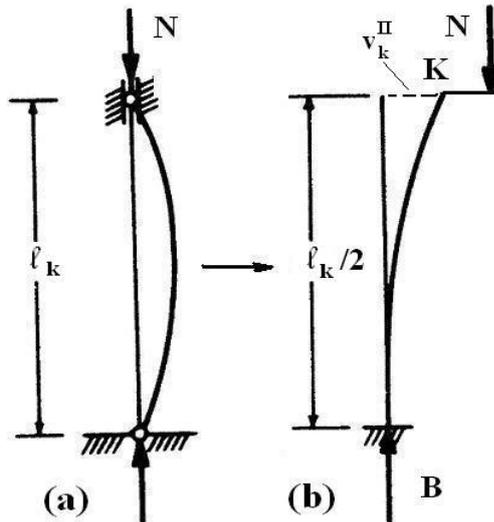


Fig. 2: Pin-jointed (a) and equivalent cantilever column (b)

In the following the differential equation governing the deflection “ y ” of a (RFC) in Fig.3 is derived. Thus, the deflections with respect to second-order theory can be calculated. In Fig.3 the deflections of a cantilever column are illustrated. The deflections denotes to the second-order theory for which the following equations are introduced. At the fixed-end support the bending moment is given by Eq. 4.1

$$M_B^II = M_B^I + N.v^II \quad (4.1)$$

The fundamental bending moment equation is

$$M(x) = N(e + v^II - y) \quad (4.2)$$

$$y'' = \frac{M(x)}{EI_m} \quad (4.3)$$

In Eq. 4.3 left wise curvature is positive. Here the EI_m is the mean stiffness of the column which is calculated in Eq. 4.4.

$$EI_m = \frac{EI_K + EI_B}{2} \quad (4.4)$$

From Eq. 4.3 it can be derived the Eq. 4.5

$$u'' + \lambda^2 u = 0 \quad (4.5)$$

where it applies

$$u = y - e - v^II \quad (4.6.1)$$

$$y = u + e + v^II \quad (4.6.2)$$

$$u'' = y'' \quad (4.6.3)$$

$$\lambda = \sqrt{\frac{N}{EI_m}} \quad (4.6.4)$$

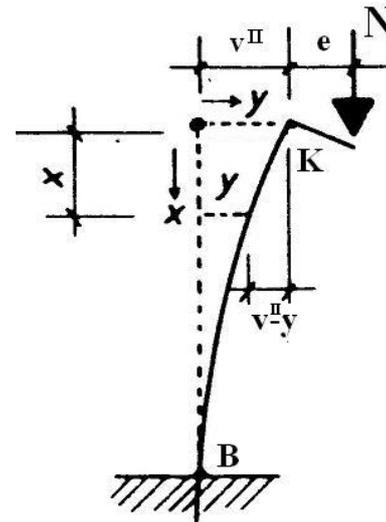


Fig.3: Statical Data for a deformed (RFC)

Taking into account the end conditions, the differential Eq. 4.5 leads to an explicit solution as given in Eq. 4.7. This equation provides the second order deflections of the (RFC) in a very simple way, by this the calculations can be carried out even with hand calculators (Hamann, 1982).

$$v^II = e \cdot \frac{1 - \cos(\lambda.l)}{\cos(\lambda.l)} \quad (4.7)$$

In Eq. 4.6.4 the stiffness can be read from Tables such as given in Table 1. The Eq. 4.7 indicates also failure state of (RFC) in fire. If the sign of $\cos(\lambda.l)$ changes a stability failure is attained. Otherwise a material failure will be present when EI attains zero in Eq. 4.6.4 due to increasing deflections and bending moments.

The fire resistance time of (RFC) according to the EC 2, 2002

The *Standard fire resistance* is defined as the ability of a (RFC) to retain bearing capacity during a standard fire exposure. DIN V ENV 1992-1-2 (Eurocode 2, 1997) gives a simple equation for the fire resistance time of (RFC) under compression loads of a non-sway reinforced concrete column. This equation is

$$R = \left(\frac{R_{eff} + R_a + R_l + R_b + R_a}{120} \right) \quad (4.8)$$

The definitions of the parameters in Eq. 4.8 can be read in EC 2.

5. CALCULATION EXAMPLES

The suggested methodology is validated by comparing its predictions of SFB-Tests with a reliable method available in the literature (Haksever, 1982).

Example I

Geometrical data of the (RFC) are given below:

$b/h = 300 / 300$ mm, rectangular cross section

$l_k = 5800$ mm, $N_d = 600$ kN

$e_0 = 0$ mm, $e_a = 5800/300 = 19.33$ mm (EC 2)

$A_{s1} = A_{s2} = 3 \Phi 20$, $c = 20$ mm

$f_{ck} = 35$ N/mm², $f_{yk} = 420$ N/mm², moisture content 4%.

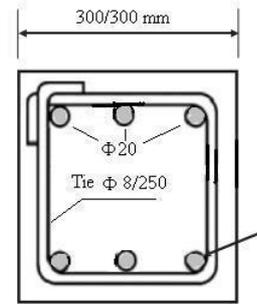


Table 2: Stiffnesses of the (RFC) in the fire

0. min. Fire Duration						
0.1	10.1	20.1	30.1	40.1	50.1	M[kNm]
						EI[Nmm²x10¹²]
19.3696	19.4303	19.4301	19.4131	19.0864	17.5804	
10. min.						
12.8861	12.8514	12.7526	12.5877	12.4508	12.1969	
20. min.						
10.0040	9.7109	9.5721	9.4799	9.4110	9.2430	
30. min.						
6.9186	6.9937	6.9299	6.8853	6.6537	6.4835	
40. min.						
3.3709	4.1806	4.2684	4.2858	4.3334	4.4030	
50. min.						
2.8392	2.8871	2.9887	3.1701	3.2797	3.3297	
60. min.						
2.7769	2.7257	2.6715	2.6293	2.6642	2.6106	
70. min.						
2.4698	2.4627	2.4718	2.4121	2.1440	2.0861	
80. min.						
2.1034	2.0416	1.7759	1.6908	1.6101	1.5328	

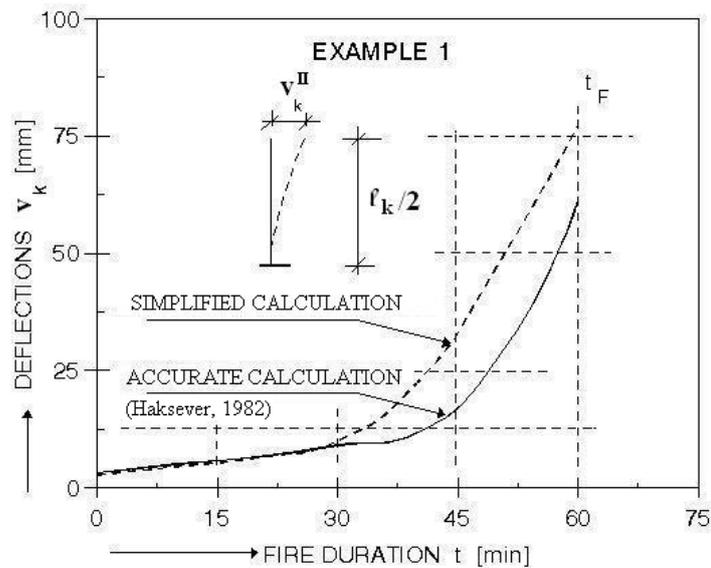
An accurate computer aided calculation of the column resulted in a fire resistance of 60 min. In Table 2, the development of the stiffness of the cross section in the standard fire is presented. With respect to the data given in this table, the simplified calculation using the Eq. 4.7 resulted in the same fire resistance. The

calculation steps are shown in Table 3 and the development of the deformations is illustrated in Fig. 4. Fig. 4 shows good agreement of the deflections and the fire resistance between the predictions of the proposed and the accurate calculation.

²The computer program can be obtained from the author.

Table 3: Calculation steps of the (RFC) in example I

Time [Min.]	Iter.	M [kNm]	EI_m [Nmm ²]	λ , Eq. 4.6.4 [1/mm]	v^{II} [mm] simpl.	V^{II} [mm] ³ accur.
0	1	11.598	0.19430E+14	0.17573E-03	2.81	3.24
	2	13.286	0.19430E+14	0.17573E-03	2.81	
10	1	13.286	0.12828E+14	0.21627E-03	4.54	5.04
	2	14.324	0.12823E+14	0.21631E-03	4.55	
20	1	14.324	0.96712E+13	0.24908E-03	6.43	6.40
	2	15.458	0.96633E+13	0.24918E-03	6.44	
30	1	15.458	0.69718E+13	0.29336E-03	9.98	9.09
	2	17.588	0.69650E+13	0.29350E-03	10.00	
40	1	17.588	0.42200E+13	0.37707E-03	22.75	11.86
	2	25.247	0.42356E+13	0.37637E-03	22.59	
50	1	25.247	0.29922E+13	0.44780E-03	52.57	27.67
	2	43.141	0.30986E+13	0.44004E-03	47.23	
	3	39.933	0.30901E+13	0.44065E-03	47.61	
60	1	39.933	0.26906E+13	0.47223E-03	77.33	61.38
	2	57.996	0.00	Material Failure		

**Fig. 4:** Development of the deflections of the column in example I during the fire**Example II**

In the following examples the stiffness table will not be given. The reader can easily produce these tables for different *moment-axial force* combinations for the fire case.

Geometrical data of the (RFC) are given below.

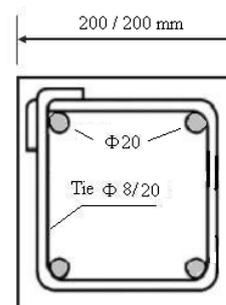
$b/h = 200 / 200$ mm, rectangular cross section

$l_k = 3500$ mm, $N = 300$ kN

$e_0 = 0$ mm, $e_a = 3500/300 = 11.66$ mm (EC 2)

$A_{s1} = A_{s2} = 2 \bar{\Phi} 20$, $c = 20$ mm

$f_{ck} = 35$ N/mm², $f_{yk} = 420$ N/mm², moisture content 4%



³ (Haksever, 1982)

Table 4: Calculation steps of the (RFC) in example II

Time [min.]	Iter.	M [kNm]	EI _m [Nmm ²]	λ, Eq. 4.6.4 [1/mm]	v ^{II} [mm], simpl.	v ^{II} [mm] acur. ³
0	1	3.498	0.38388E+13	0.27955E-03	1.55	1.79
	2	3.962	0.38389E+13	0.27955E-03	1.55	
20	1	4.286	0.17789E+13	0.41066E-03	3.83	3.83
	2	4.647	0.17792E+13	0.41062E-03	3.83	
40	1	5.246	0.10874E+13	0.52524E-03	7.57	7.39
	2	5.767	0.10874E+13	0.52524E-03	7.57	
60	1	6.305	0.81086E+12	0.60826E-03	12.38	
	2	7.212	0.79145E+12	0.61567E-03	12.96	
	3	7.386	0.78774E+12	0.61712E-03	13.08	
70	1	7.386	0.31973E+12	0.96865E-03	∞	12.41
Stability Failure; Cos(λ.l) < 0						
75	Stability Failure					56.47

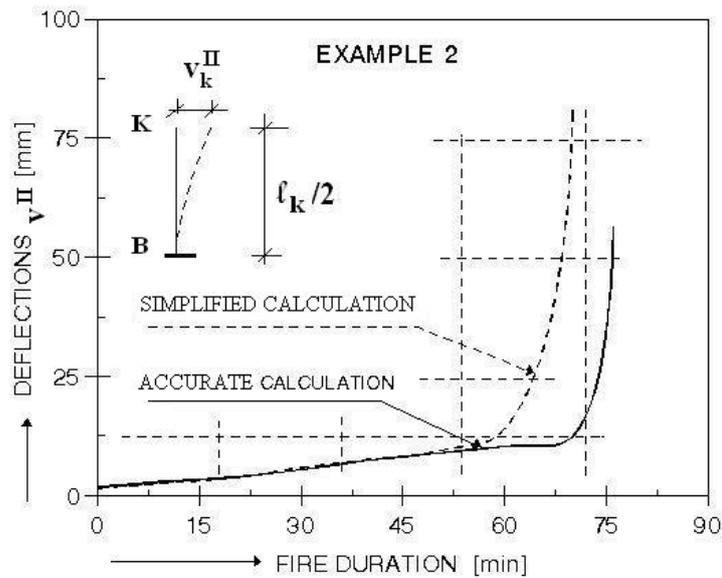


Fig. 5: Development of the deflections of the column in example II during the fire

An accurate computer aided calculation of the column resulted in a fire resistance of 75 min. Calculation of the fire resistance of (RFC) is shown in

Table 4 and the development of the deformations is illustrated in Fig. 5. A good agreement between the two methods is present also in this example II

Example III

Geometrical data of the (RFC) are given below.

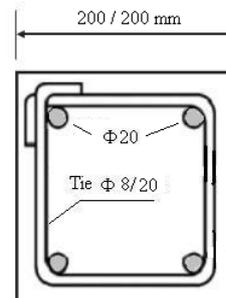
b/h = 200 / 200 mm, rectangular cross section

l_k = 2750 mm, N = 150 kN

e₀+ e_a = 100 mm

A_{s1}= A_{s2} = 2 Φ 20, c = 20 mm

f_{ck} = 35 N/mm², f_{yk} = 420 N/mm², moisture content 4%.



¹Sonderforschungsbereich 148, a special research activity for structural fire protection

An accurate computer aided calculation of the column resulted in a fire resistance of 60 min. Calculation of the fire resistance of (RFC) is shown in Table 5 and the

development of the deformations are illustrated in Fig. 6. A good agreement between the two methods is present also in this example III.

Table 5: Calculation steps of the (RFC) in example III

Time [Min.]	Iter.	M [kNm]	EI _m [Nmm ²]	λ, Eq. 4.6.4 [1/mm]	v ^{II} [mm] simpl.	v ^{II} [mm] accur. ³
0	1	45.000	0.12829E+14	0.24512E-03	9.78	9.93
	2	47.934	0.12606E+14	0.15427E-03	9.96	
	3	47.989	0.12602E+14	0.15429E-03	9.97	
20	1	48.705	0.80703E+13	0.19280E-03	16.06	16.01
	2	49.819	0.80677E+13	0.19283E-03	16.07	
40	1	51.371	0.44921E+13	0.25843E-03	31.05	26.85
	2	54.315	0.44741E+13	0.25895E-03	31.20	
60	1	57.979	0.28282E+13	0.32569E-03	54.83	66.03
	2	61.449	0.28226E+13	0.32601E-03	54.97	
70	1	61.449	0.24132E+13	0.35258E-03	67.76	Material Failure
	2	65.329	0.23708E+13	0.35572E-03	69.44	
	3	65.831	0.23653E+13	0.35613E-03	69.66	
80	1	65.831	0.17258E+13	0.41693E-03	111.15	
	2	78.345	0.10581E+13	0.53248E-03	292.18	
	3	132.27	0.00	Material Failure		

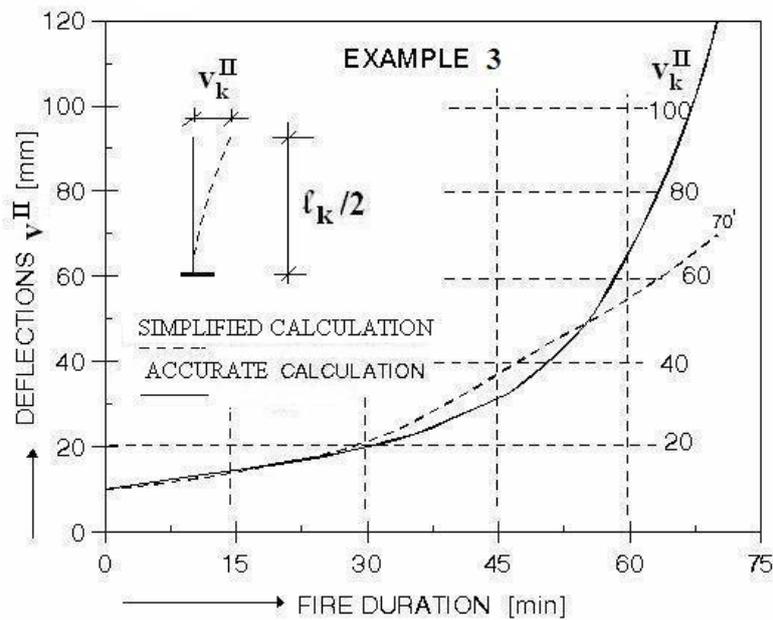


Fig. 6: Development of the deflections of the column in example III during the fire

Example IV

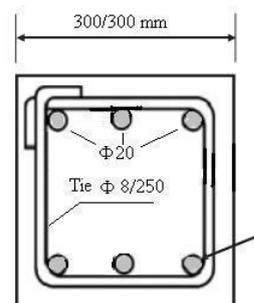
Geometrical data of the (RFC) are given below.

b/h = 300 / 300 mm, rectangular cross section

l_k = 4600 mm, N = 300 KN, e₀+e_a = 150 mm,

A_{s1} = A_{s2} = 3 Φ 20, c = 20 mm

f_{ck} = 35 N/mm², f_{yk} = 420 N/mm², moisture content 4%.



An accurate computer-aided calculation of the column resulted in a fire resistance of 70 min. Calculation of the fire resistance of (RFC) is shown in Table 6 and the

development of the deformations is illustrated in Fig. 7. A good agreement between the two methods is present also in this example IV.

Table 6: Calculation steps of the (RFC) in example IV

Time [Min.]	Iter.	M [kNm]	EI _m [Nmm ²]	λ, Eq. 4.6.4 [1/mm]	v ^{II} [mm] simpl.	v ^{II} [mm] accur ³
0	1	15.000	0.24965E+13	0.24512E-03	5.96	6.13
	2	15.894	0.24729E+13	0.24629E-03	6.02	
	3	15.903	0.24727E+13	0.24630E-03	6.02	
20	1	16.079	0.15763E+13	0.30848E-03	9.72	9.59
	2	16.458	0.15757E+13	0.30853E-03	9.73	
40	1	16.944	0.10488E+13	0.37818E-03	15.23	14.94
	2	17.285	0.10479E+13	0.37834E-03	15.25	
60	1	17.587	0.78276E+12	0.43776E-03	21.32	33.17 Stability Failure
	2	18.198	0.77174E+12	0.44087E-03	21.68	
	3	18.252	0.77076E+12	0.44115E-03	21.71	
70	1	18.252	0.10277E+12	0.12081E-02	∞	Stability Failure; Cos(λ.l) < 0

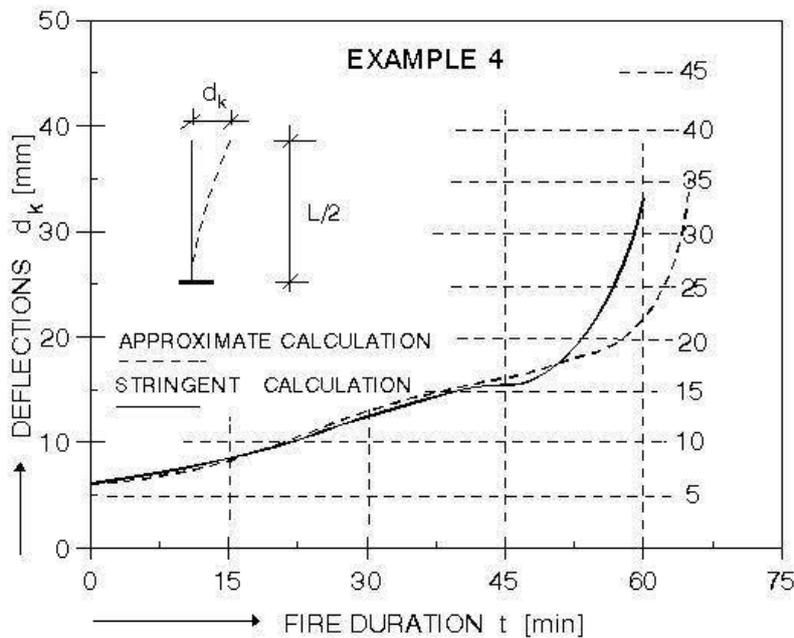


Fig. 7: Development of the deflections of the column in example IV during the fire

6. COMPARISON OF THE METHODS WITH REGARD TO THE FIRE RESISTANCE

The capability of the method the European standard - (EC2) - for assessing the fire resistance time of reinforced concrete columns will be discussed in this chapter and the results will be compared with the method presented in this paper.

According to (EC2), the fire resistance time of a reinforced concrete column is given by Eq. 4.1 The related procedure requires the second-order effects to be included. (EC2) assumes that the (RFC) is not insulated.

That is also the case in the examples given above. The results of the analysis are given in Table 7. The last two columns show the calculated fire resistance according EC2 and the accurate calculation. Obviously the fire resistance predicted by (EC2) is more than the resistance time calculated both with accurate and the simplified method in this paper. A similar conservativeness of (EC2) with regard to the resistance time of circular reinforced concrete columns was reported by (Franssen et al., 2003) and (Bratina et al., 2005).

¹Sonderforschungsbereich 148, a special research activity for structural fire protection

Table 7: Fire resistance of the (RFC) according to (EC2) and f_t (Haksever, 1982).

Example	$N_{Ed,fi}$ [kN]	N_{Rd}	R_b [mm]	R_t [m]	R_a [mm]	$R_{\eta fi}$	R [Min.]	t_F
1	600.	888.	27.00	-7.68	128.00	26.92	265.0	62.
2	300.	464.	18.00	14.40	48.00	29.29	102.0	75
3	150.	189.	18.00	21.60	48.00	17.13	94.0	61.0
4	300.	382.	27.00	3.84	128.00	17.82	271.0	65.0

The numerical results for the resistance time with the predictions of the European building code (EC2, 1997) make it necessary the Eq. 4.8 to be modified. (EC2, 1997) predicts much longer resistance time than calculated in this paper although the standards are assumed to give the safe side results.

The equation in (EC2, 1997) is modified as given below:

$$\mu_G = \frac{N_{Ed,fi}}{N_{Rd}} \tag{6.1}$$

$$R_{\eta fi} = 180 \cdot \mu_G \cdot (1 + \omega) \tag{6.2}$$

$$R = \left(\frac{R_{\eta fi} + R_a + R_t + R_b + R_n}{120} \right)^{0.7} \tag{6.3}$$

The other expressions in (EC2, 1997) are taken over unchanged. The new results are given in Table 8 (for ω s. notations).

Table 8: Fire resistance of the (RFC) according to the modified (EC2, 1997)

Example	$N_{Ed,fi}$ [kN]	N_{Rd}	R_b [mm]	R_t [m]	R_a [mm]	$R_{\eta fi}$	R [Min.]	t_F
1	600.	888.	27.00	-7.68	128.00	121.96	56.	62.
2	300.	464.	18.00	14.40	48.00	173.81	52.	75.
3	150.	189.	18.00	21.60	48.00	213.07	58.	61.
4	300.	382.	27.00	3.84	128.00	187.70	66.	65.

Apparently the modified new equation gives better results than the Eq. 4.8 in (EC2, 1997). This observation can also be seen in Fig. 8, although in example 2 there is a disagreement between the Eq. 4.8 and accurate

calculation. The Eq. 4.8 may be useful to calculate the fire resistance of (RFC) with fixed supports. However, some further systematic analyses could resolve the differences in the calculation results if need be.

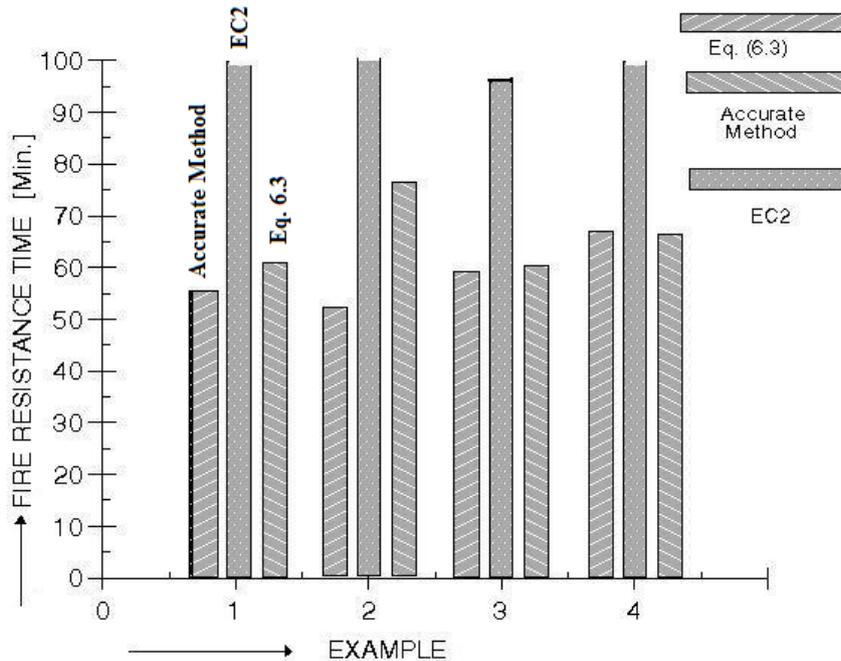


Fig. 8: Calculation results of the examples for EC2

7. SUMMARY

The numerical determination of the fire resistance time of reinforced concrete columns leads to comparatively extensive computer calculations. In this paper a simplified calculation method for (RFC) under fire exposure is presented. The method enables to determinations of the deflections as well as the fire resistance time of (RFC) by the structural engineer without efficient computer aid. For this purpose stiffness tables must be prepared beforehand (Haksever, 1978). The reader can easily produce these tables for different M, N and reinforcement combinations of rectangular concrete cross sections and keep them ready for further calculations. The deflections and the fire resistance of the (RFC) can be determined by means of a simple calculation even using a hand calculator. Because the deflections must be determined with respect to the second-order theory it is necessary to do a few iteration steps in order to obtain the final deflections. The steps can be chosen at first with longer intervals, but, in the near of collapse state of the (RFC) these intervals must be reduced suitably in order to determine the fire resistance time as realistically as possible. Some calculation examples are shown in chapter 5.

NOTATIONS

A_c	Area of the concrete cross-section	[mm ²]
A_s	Area of the reinforcements	[m ²]
C_p	Specific heat capacity	[J/kgK]
c	Concrete cover	[mm]
d	Diameter of reinforcement	[mm]
e	Eccentricity	[mm]
e_a	Imperfection in eccentricity	[mm]
f	Strength	[N/ mm ²]
k	Heat conductivity	[W/mK]
l	Length	[mm]
l_k	Buckling length	[mm]
M	Bending moment	[kNm]
N_{d_s}	Applied axial load	[kN]
$N_{Ed,fb}$	Design load of axial force, (EC2)	[kN]
N_{Rd}	Axial load bearing capacity, (EC2)	[kN]
RFC	Reinforced concrete member(s)	
T	Temperature	[K]
t	Time	[sec]
t_F	Fire resistance	[min]
x, y	Place co-ordinates	[mm]
y_c	The height of centroid of a cross section	[mm]

Additional Symbols

$\alpha = k / (\rho C_p)$	Thermal diffusivity of the material	[m ² / s]
$\bar{\Phi}$	Diameter of reinforcements	[mm]
ε	Strain	
κ	Curvature	[1/m]
$\omega = \frac{A_s \cdot f_{yd}}{A_c \cdot f_{cd}}$	Reinforcement ratio	
ρ	Density	[kg/ m ³]

The other notations are defined where they appear in the text.

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